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13. ABSTRACT (Maximum 200 words)
The interaction of atmospheric turbulence with light rays leads to serious degradations to images taken through the atmosphere. This research project was concerned with the development of rigorous mathematical models, computational algorithms and high performance computer implementations, in the optical imaging areas of adaptive-optics computations and image post-processing by reconstruction and other enhancement techniques. The work has applications in defense, including the airborne laser weapons program, and to civilian technology, including astronomical, commercial, and medical imaging. An Air Force IBM SP2 supercomputer with 576 nodes was used for parallel algorithm software development. Activities over the period of the grant included three trips each year to Phillips Air Force Laboratory (PLK), including visits to the associated Starfire Optical Range. Eighteen papers were completed and twenty-eight presentations made. Close collaboration was held with researchers at PLK. Much of the motivation for this work came from interactions with Dr. Brent Ellerbroek at the Starfire Optical Range.

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FINAL REPORT

1 May 1994 to 30 April 1997

Computations in Imaging and Related Problems

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1 Introduction

Applied as the image of an object is formed, *adaptive optics* techniques compensate for degradations added along the path of the light from the object being imaged. *Image restoration* (post-processing) tools are then used to scrub the captured optical image even cleaner. The first phase is a massive control problem. The second is a delicate inverse problem. Both adaptive optics and image restoration demand *sophisticated mathematics and state-of-the-art computation.*

Image restoration involves the removal or minimization of degradation (blur, clutter, noise, etc.) in an image using a priori knowledge about the degradation phenomena. Blind restoration is the process of estimating both the true image and the blurring operator from the degraded image characteristics, using only partial information about degradation sources and the imaging system. Our main interest concerned optical image enhancement, where the degradation involves a convolution process. Image restoration techniques are also providing clearer views of objects.

The power of these tools is substantial. One of our simulations, for example, shows them improving the resolution of a telescope from being barely able to spot an object the size of a house trailer in earth's orbit to detecting a hand waving from the trailer's window!

Testifying before two committees in Congress last year in support of the Department of Defense's basic research program, Principal Investigator (PI) Plemmons outlined the varied uses of adaptive optics and image restoration tools: "Our DoD research projects are driven in part by the satellite imaging, identification, and classification program, and especially the need for high technology missile defense systems. Applications to civilian technology include astronomical and medical imaging... fluorescence microscopy in three dimensions... deblurring images of the retina through the eyeball, [and] remote sensing images of the earth for agricultural, law enforcement, and geophysics purposes."

One goal of the research was to develop fast algorithms that operate in near real-time. Had sufficiently sophisticated hardware and software been available, for example, the American pilot shot down two years ago in Bosnia might have avoided the ground-based missile fired at him; the system would have identified the missile threat from a satellite and then relayed the information directly to defense systems on the pilot's aircraft. In other settings, such as a medical office or a criminal investigation, optical postprocessing

can be more leisurely, but time is still at a premium.

Adaptive optics compensation plays an essential role in current state-of-the-art atmospheric telescope imaging technology. The ideal earth-based astronomical telescope is built on bedrock, high on a remote mountain. The solid foundation partially stabilizes the telescope against wind and other potential causes of vibration, while the altitude and isolation minimize atmospheric degradation. The Hubble space telescope carries this logic to its natural extreme, but even the Hubble's accuracy is limited by the effects of thermal stresses and other forces that shift the phase of the incoming light ever so slightly.

Ideally, light from a distant object high above the earth's atmosphere arrives at a telescope's mirror as a single planar wavefront. The only limit on resolution should be diffraction by the telescope mirror aperture. In imaging through the atmosphere, tiny local variations in the index of refraction of the atmosphere induce small phase errors that make the incoming plane wave look more like a sheet of crumpled paper. The mirror then adds phase errors of its own; even a theoretically perfect mirror will be distorted by thermal stresses, not to mention the effects of small vibrations in the telescope structure.

In this setting, active and adaptive optics attempt to compensate for these phase errors using as a reference the phase error in the image of a guide star, either a bright natural star very near the target image or a "star" created by directing a laser into the atmosphere. Guide stars are especially effective against the degradation by atmospheric turbulence of images collected by ground-based telescopes because they provide an estimate of the unknown blurring operator. Furthermore, thermal distortion and gravity can induce small deformations in lightweight mirrors.

Active optics corrects these very low frequency errors by delicately nudging the primary mirror with hydraulic actuators. Adaptive optics corrects the higher frequency errors caused by atmospheric irregularities and telescope vibration. The distortion measured using the guide star drives a control system that adjusts a separate set of mirrors. (The primary mirror is too big to respond fast enough.)

With adaptive optics, instruments like the 3.5-m telescope at the Starfire Optical Range of the U.S. Air Force Philips Laboratory in New Mexico, can partially correct the image before it is recorded. Note that this real-time control requires extraordinarily high-speed computation – up to 10 billion floating point operations per second.

Postprocessing further restores the adaptive optics recorded image to a state even closer to perfection by filtering out any remaining noise and blur that can be distinguished from the image. The classic tool is regularized least squares; one of the newest techniques we investigated is based on the solution of a nonlinear partial differential equation. Like adaptive optics, both demand cutting-edge computation.

1.1 Adaptive Control of Deformable Mirrors: Acquiring the Image

Many modern astronomical telescopes are now built with deformable mirrors that can be adjusted dynamically. Real-time control of the separate actuators of such a mirror can accommodate distinctly different sources of error, such as wind shake and time-varying atmospheric distortion. Each source has its own characteristic temporal frequency; those of wind shake, for example, are typically much higher than those of atmospheric turbulence.

The key to adjusting the mirror actuators on the fly is to choose a basis set of mirror deformations, known as mirror control modes, which best control each disturbance at a bandwidth matched to its characteristic frequency. In contrast, correcting all disturbances at a common bandwidth will either allow high frequency errors to sneak through if this bandwidth is set too low, or add unnecessary noise to the correction of low frequency errors if the bandwidth is too high.

Brent Ellerbroek of Philips Laboratory, and PI Plemmons have developed a multiple bandwidth modal control strategy that can minimize mean squared phase error in the image across multiple sources of error (see publications [1,9,17]). These technique advances previous approaches by enabling the simultaneous optimization of both the mirror control modes and the associated control bandwidths without choosing in advance the basis set of control modes. The optical performance of the system at a particular bandwidth is characterized by a matrix. The optimal control modes could be determined by finding the one unitary transformation that comes closest to simultaneously diagonalizing all of these optical performance matrices.

However, the approximate simultaneous diagonalization of more than two matrices is not an easy task to formulate algorithmically, and any such scheme would be computationally expensive. Instead, we use a novel trace maximization approach based on a hill climbing scheme relative to pairs of matrices

in order to minimize mean squared phase error.

In particular, we have studied in publications [1] and [17] a non–smooth optimization problem arising in adaptive optics, which involves the optimal real-time control of deformable mirrors designed to compensate for atmospheric turbulence and other image degradation factors, such as wind–induced telescope vibration. The surface shape of this mirror must change rapidly to correct for time–varying optical distortions caused by these sources of image degradation. One formulation of this problem yields a functional

$$f(U) = \sum_{i=1}^{n} \max_{j}\{(U^T M_j U)_{ii}\}$$

to be maximized over orthogonal matrices U, where U and a fixed collection of $n \times n$ symmetric matrices M_j. We consider the situation which can arise in practical applications where the matrices M_j are "nearly" pairwise commutative. Besides giving useful bounds, results for this case lead to a simple corollary providing a theoretical closed–form solution for globally maximizing f if the M_j are simultaneously diagonalizable. However, even here conventional optimization methods for maximizing f are not practical in this real-time environment. The general optimization problem is quite difficult and is approached using a heuristic Jacobi-like algorithm. Numerical tests using the algorithm indicated that the performance of adaptive optics systems, such as those of interest to the Air Force, can be improved by the use of our Jacobi-like algorithm.

This scheme can show substantial improvement over single bandwidth control. It lends itself to parallel implementation and nearly real-time computing (see publication [17]).

1.2 Image Postprocessing: Cleaning Up the Image

The techniques of adaptive optics are helping ground-based sensors to capture better, but hardly perfect, astronomical images. More mundane devices like surveillance cameras are cursed from the start with gritty, low-resolution images. In both settings, image restoration techniques can help obtain a clearer picture by separating the image from the degradations.

Classic restoration techniques often model the received image as the sum of noise and a blurring operator acting on the true image. (The blurring operator is a convolution with the point-spread function that characterizes

the aberrations.) These techniques seek to recover the correct image by solving an ill-conditioned least squares problem, usually regularized through the addition of some smoothness requirement to be satisfied by the restored image.

This linear problem is a formidable computation because of the size of the blurring operator – its dimension is the number of pixels in the image – and its inherent ill-conditioning. Regularization can render the restoration problem solvable in practice, by neutralizing some of the ill-conditioning, but it also removes sharp edges and similar distinguishing features. Once distinctive characteristics, for example, can become unrecognizable.

Leonid Rudin of Cognitech, Inc., and Stanley Osher of UCLA, formerly of Cognitech, have developed a widely used alternative known as the total variation (TV) technique because it minimizes the total variation of the image instead of its second derivative. Rather than insisting on a completely smooth image, TV requires only that it have bounded variation, permitting sharp edges but eliminating spurious oscillations. They came to their approach in part through methods used for tracking shock fronts in gas dynamics calculations, a setting that also seeks to preserve sharp boundaries without introducing extraneous detail.

The problem at the heart of a TV image restoration problem is equivalent to a nonlinear partial differential equation (the Euler equation of the constrained minimum variation problem). The restored image solves this steady-state problem. In their original work, Rudin and Osher found that steady-state solution by iterating in time from an appropriate initial condition. More recently, Tony Chan of UCLA, Curt Vogel of Montana State University, and PI Plemmons, have proposed preconditioned conjugate gradient methods for iterating to the solution, at a much faster rate.

The down side of TV-based image restoration is its computational expense. For example, Rudin and Osher's time-stepping scheme for solving the TV differential equation can be slowed by stability restrictions that force it to take relatively small time steps.

In this respect, PI Plemmons and his colleagues James Nagy of Southern Methodist University, Paul Pauca of Duke University, and Todd Torgersen of Wake Forest have proposed a compromise: use TV methods to sharpen the estimate of the blurring operator obtained from an auxiliary source like a guide star, then apply quicker linear restoration algorithms to the image (see publications [14] and [18]).

Since the blurring operator is localized, TV techniques can be applied to it

fairly cheaply, thereby gaining from the ability of minimum total variation to preserve sharp transitions without the expense of a complete TV restoration. Using the TV estimate of the blurring operator, linear restoration is then applied adaptively to subregions of the full image allowing subregions of the image to converge at a more natural rate, a technique the authors call the *space-varying regularization* (SVR) technique. This novel SVR method reduces the possibility of excessive smoothing and magnification of noise during the linear restoration phase.

PI Plemmons and his colleagues solve the linear restoration subproblems iteratively using SVR. The challenge is to continue the iterations long enough to amplify the components associated with the image but not so long that the noise is amplified as well. By simultaneously monitoring the size of the image components and the residual, or equation error, they can choose a stopping criterion appropriate for the portion of the image under study. This approach stops iterations early for a region with little spatial variation, such as an image of empty sky, but lets them run longer for a busier region that includes, say, a piece of the satellite or star under observation.

Simulations using the 3.5-m telescope at the Starfire Optical Range show impressive improvements: combined with the multiple-bandwidth adaptive optics control of Ellerbroek, Plemmons, and others, use of SVR strengthens resolution by a factor of about 50. A telescope that can discern nothing smaller than 30 meters at a range of 1000 km using its optics alone finds its resolution improved to 20 *centi*meters when a combination of adaptive optics and SVR postprocessing is used.

Many image restoration problems fall into the broad category of blind deconvolution because it is necessary to estimate both the true image as well as the blur from the degraded image using only partial information about the blurring operator. Blind deconvolution approaches include those developed by Julian Christou of Phillips Laboratory and by Plemmons and his colleagues Michael Ng of the Australian National University and Senzheng Qiao of McMaster University (see publication [16]). These particular approaches couple constrained optimization with nonlinear conjugate gradient methods. A major part of the PI's recent image postprocessing work has been concerned with developing new, effective blind deconvolution methods.

We have also investigated a projection-based conjugate gradient technique with nonuniform regularization for multichannel (multiframe) restoration of and object. Data corresponding to multiple point spread functions was also taken into account. Numerical results for sequential as well as parallel im-

plementations on the Air Force IBM SP2 at the Maui High Performance Computing Center are given. The paper containing the results from this recent study is currently being finalized.

2 Objectives

The objectives of this research project included the development of rigorous mathematical models, computational algorithms and high performance computer implementations. Specifically, the project was concerned with the areas of on-line optimization computations for controlling deformable mirrors, and corresponding off-line computations in image reconstruction and restoration. When an otherwise collimated, coherent beam of light encounters a turbulent flow field that includes density fluctuations, its optical wavefront becomes aberrated causing the beam to be degraded. The interaction of the fluid with the light is termed "Aero-Optics." Our work to enhance the quality of images has applications in defense, including the *air borne laser weapons program* (ABL), and to civilian technology, including astronomical and medical imaging.

The objectives of the adaptive optics phase of the AFOSR project were to conduct rigorous mathematical research in the aero-optics areas of adaptive closed-loop deformable mirror control and image reconstruction. Parallelizations of the computational algorithms were investigated and implemented on the Air Force massively parallel SP2 at the Maui High Performance Computing Center. Computations in adaptive-optics research is being continued, in collaboration with Dr. Brent Ellerbroek at the Air Force Phillips Laboratory, on the topic of optimal closed-loop real-time deformable mirror control. Our recent research was expanded to include 1) improved optimization algorithms (see publications [9] and [17]), 2) methods for updating computations for the imaging system parameters, in collaboration with Brent Ellerbroek and also Moody Chu at NC State University, and 3) the investigation of data-massive computations necessary for segmented mirrors with very many degrees of freedom.

Also, iterative restoration methods are being studied with the purpose of obtaining algorithms that are both computationally efficient and stable (see publications [3-6], [11-15], and [18]). Work was expanded to include blind deconvolution, which is of particular importance in aero-optics applications (see publication [16]), multiframe restoration and the treatment of spatially varying blur.

Research collaboration on these projects has continued with researchers at Phillips Laboratory, Kirtland AFB, and at AFIT, Wright-Patterson AFB.

3 Accomplishments/New Findings

The research was concerned with major projects in astro-imaging. One project concerns deformable mirror adaptive control studies in collaboration with Dr. Brent Ellerbroek at the Air Force Phillips Laboratory Starfire Optical Range. Specially designed deformable mirrors operating in a closed-loop adaptive-optics system can partially compensate for the effects of atmospheric turbulence. The systems detect the distortions using either a natural guide star (point) image or a guide star artificially generated from the back scatter of a laser generated beacon. We have used the massively parallel IBM Scalable POWERparallel 2 (SP2) at the Air Force Maui High Performance Computing Center (MHPCC) for simulation studies in order to better understand the characteristics of our algorithms.

A second project concerned nonlinear iterative methods for image post-processing computations. We further enhance restored image quality of our restorations by use of nonlinear optimization methods, blind deconvolution, and adaptive space-varying restoration using variational segmentation methods.

The aberration of light caused by the Earth's atmosphere has been known to be the limiting factor in aero-optics observations since at least 1730 when Sir Isaac Newton mentioned it in his book "Opticks". These aberrations are a deformation of the incoming optical wavefront. The aberrations have two major effects which limit observations through the atmosphere:

- The limiting of the angular resolution of telescopes to about 1 arc second.

- A reduction in the central power density of the image.

The limit on the angular resolution means that a multi-million pound 8 meter telescope without adaptive-optics has the same resolution of a 15cm amateur telescope. The usual method now employed to correct for this problem is to use a deformable mirror. Light entering a telescope is split into two parts: one forms the image and the other goes to a wavefront sensor. The wavefront information is then processed and piezo-electric crystals are used to deform a mirror in the correct manner that acts as the corrective element. Figure 1 gives an overall diagram of a typical adaptive-optics (AO) closed-loop control system.

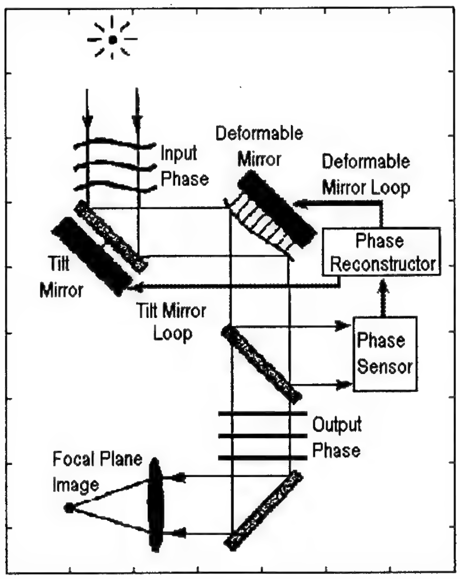

Figure 1: Typical AO System

In a paper which appeared in the J. Optical Soc. Amer. A, the PI has helped to develop a theory with applications to closed-loop adaptive control methods to adjust the shape of these mirrors in real-time. A second paper, referenced as [17] in the Publications Section, will appear in the SIAM J. on Optimization. Numerical tests using this algorithm indicate that in the presence of windshake jitter, the performance of a closed-loop adaptive-optics system can be improved by the selection of distinct and independently optimized control bandwidths for separate modes of the wavefront distortion profile. These results may be relevant, for example, for the adaptive-optics system planned for the 8-meter Gemini-North telescope to be located on the mountain of Mauna Kea in Hawaii, where both windshake and atmospheric turbulence effects must be compensated to achieve the desired levels of optical system performance.

Furthermore, our efforts for improvement in optical image quality is attempted in two steps, forming a hybrid method. The first step occurs as the observed image is initially formed, as described above. The second stage of compensating for the effects of atmospheric turbulence occurs off-line, and consists of the processing step of *image restoration.* An image partially corrected by the adaptive-optics procedure discussed above can generally be

enhanced further by off-line computer image restoration. Our work here concerns preconditioned iterative methods for the solution of certain large ill-posed inverse problems, where the solution does not depend continuously on the data. The image formation process can be modeled as:

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y;s,t)f(s,t)dsdt + \eta(x,y)$$

where $g(x,y)$ is the observed (degraded) image, $f(x,y)$ is the true (original) image (unknown), and $\eta(x,y)$ is assumed to be additive, Gaussian noise. Here, $h(x,y;s,t)$ represents the blurring point spread function (PSF). Applications abound in science and engineering. Our particular application concerns image restoration computations. Papers [3-6], [8], [11], [13-15] and [18], referenced in the Publications Section, report some of our recent post-processing work. The interest is on the development of fast algorithms and their extension to iterative blind deconvolution, where the blurring operator as well as the image is to be estimated. Our work in [14] and [18] concerned a new space-varying regularization approach, and associated techniques for accelerating the convergence of iterative image postprocessing computations. Denoising methods, including total variation minimization, followed by segmentation-based preconditioning methods for minimum residual conjugate gradient iterations, were investigated. Regularization is accomplished by segmenting the image into (smooth) segments and varying the preconditioners across the segments. The method appears to work especially well on images that are piecewise smooth. Our algorithm has computational complexity of only $O(\ell n^2 \log n)$, where n^2 is the number of pixels in the image and ℓ is the number of segments used. Also, parallelization is straightforward. Numerical tests are reported on both simulated and actual atmospheric imaging problems. Comparisons were made with the case where segmentation is not used. It was found that our approach is especially attractive for restoring images with low signal-to-noise ratios, and that magnification of noise is effectively suppressed in the iterations, leading to a numerically efficient and robust regularized iterative restoration algorithm.

Our variational image restoration method used in publication [18] is described next. If we let $\mathcal{H}$ denote the blurring operator and η the noise process, then the image restoration problem with additive noise can be expressed as a linear operator equation

$$g = \mathcal{H}f + \eta, \tag{1}$$

where g and the unknown f denote functions containing the information of the recorded and original images, respectively. Note that when $\mathcal{H} = \mathcal{I}$, the identity operator, the image restoration problem means to extract the image f from a noisy image g. This problem is usually referred to as the *denoising* problem.

Let u and v denote two-dimensional variables. If $\mathcal{H}$ is a convolution operator, as is often the case in optical imaging, then the operator acts uniformly (i.e., in a spatially invariant manner) on f. Here, (1) can be written as

$$\mathcal{H}f(u) = \int_\Omega h(u-v)f(v)dv. \tag{2}$$

The problem is to both deconvolve and denoise the recorded image during the reconstruction process, and we refer to this as the *denoising and deblurring* problem. In optical imaging, the kernel h in (2) is called the convolution *point spread function* (PSF). The Fourier transform of h is called the *optical transfer function* (OTF). After discretization of (1), the spatial operator $\mathcal{H}$ defined by h in (2) is a matrix that we denote by H. Here, in the spatially invariant case, H is a *block Toeplitz matrix with Toeplitz blocks.* Thus the fast Fourier transform (FFT) can be used in computations involving H.

A classical approach employed for solving (1) is that of penalized least squares, which is also called Tikhonov regularization in the inverse problems literature. This requires minimization of the expression

$$\|\mathcal{H}f - g\|^2 + \alpha J(f), \tag{3}$$

where $\|\cdot\|$ denotes the norm on $L^2(\Omega)$, α is a positive (regularization) parameter and the functional $J(f)$ serves the purpose of stabilizing the least squares problem and penalizing certain undesirable artifacts like spurious oscillations in the computed f. Various choices of $J(f)$ can be made, including $\|Sf\|^2$, where S is some smoothing differential operator, or the identity. This model leads to fast linear methods for computing f, and is often the method of choice by practitioners. However the use of other norms such as the $\mathcal{L}_1$ norm, lead to nonlinear minimization methods which sometimes result in superior enhancement of blocky, noisy images, but with added computational cost. Such approaches are described next.

We use an image enhancement method based on solving a nonlinear PDE constrained minimization problem where the function being minimized is the Total Variation (TV) of the image $f = f(x, y)$. We consider the following

constrained minimization problem:

$$\min_f \int_\Omega |\nabla f| du \qquad \text{subject to} \;\; \|\mathcal{H}f - g\|^2 = \sigma^2, \tag{4}$$

where ∇f denotes the gradient of f, and σ is the noise level. At a point $u = (x, y)$ in the image domain, $f(u) = f(x, y)$, and so

$$|\nabla f(u)| = \sqrt{f_x^2 + f_y^2}. \tag{5}$$

The quantity

$$\int_\Omega |\nabla f| du = \int_\Omega \sqrt{f_x^2(x,y) + f_y^2(x,y)}dxdy \tag{6}$$

is called the *total variation* norm of f. The minimization in (4) is a form of regularization, a step necessary in solving most ill-posed inverse problems. The TV method is especially effective for recovering a blocky, discontinuous, function from noisy data.

Consider the following closely-related Tikhonov regularization problem (3), where

$$\alpha J(f) = \alpha \int_\Omega |\nabla f| du. \tag{7}$$

Here α is a positive regularization parameter which measures the trade-off between a good fit and an oscillatory solution. This method corresponds to the use of the $\mathcal{L}_1$ norm in the discrete case. At a stationary point, the gradient of (3) vanishes, giving:

$$z(f) \equiv \mathcal{H}^*(\mathcal{H}f - g) - \alpha \nabla \cdot \left(\frac{\nabla f}{|\nabla f|} \right) = 0, \qquad u = (x, y) \in \Omega. \tag{8}$$

Due to the term $1/|\nabla f|$, (8) is a degenerate nonlinear second order diffusion equation. The degeneracy can be alleviated by modifying the diffusion coefficient. More precisely, let $\hat{f}$ be an approximation to f given by

$$\kappa_\beta(f) = \frac{1}{\sqrt{|\nabla f|^2 + \beta}} \qquad \beta > 0, \tag{9}$$

$$\mathcal{L}_\beta(f)\hat{f} = -\nabla \cdot (\kappa_\beta(f) \nabla \hat{f}), \tag{10}$$

and define

$$\mathcal{R}_\beta(f)\hat{f} = (\mathcal{H}^*\mathcal{H} + \alpha \mathcal{L}_\beta(f))\hat{f}. \tag{11}$$

Then (8) becomes the following non-degenerate system

$$\mathcal{R}_\beta(f)\hat{f} = \mathcal{H}^* g, \quad u = (x, y) \in \Omega, \quad \text{with} \qquad u = (x, y) \in \partial\Omega. \qquad (12)$$

Various numerical schemes have been devised to obtain the minimizer of the functional (7). For example Rudin and Osher suggested an explicit time marching scheme. However, the time step must be chosen small. Thus the number of iterations to optimal convergence can be quite large. Vogel and Oman introduced a *lagged diffusivity fixed point iteration* approach, which we denote by FP, to solve (12). If $R_\beta(f^k)$, H and L_β denote the discretization matrices of $\mathcal{R}_\beta(f^k)$, $\mathcal{H}$ and $\mathcal{L}_\beta$, respectively, then the FP iteration will produce a sequence of approximations $\{f^k\}$ to the solution f and can be expressed as a sequence of systems of linear equations:

$$R_\beta(f^k)f^{k+1} \equiv (H^*H + \alpha L_\beta(f^k))f^{k+1} = H^* g, \qquad k = 0, 1, ... \qquad (13)$$

In the denoising case, numerical experiments cited by Vogel and Oman indicated that the FP iteration method often gives a faster convergence rate than the time marching method, with overall greater speed for the entire process. Note that in (13), obtaining f^{k+1} from f^k requires one to solve a large linear system with coefficient matrix $H^*H + \alpha L_\beta(f^k)$. For deconvolution, H is block Toeplitz with Toeplitz blocks. In any case, the matrix $\alpha L_\beta(f^k)$ is a 2-D nonconstant Laplacian with five bands. Its spectrum can vary widely over the outer iterations, i.e. with the index k.

A disadvantage of experimentally obtained data sets representing the PSFs (obtained, for example, using guide stars) is that they are also subject to degradations caused by noise during the image formation process. Thus, removal of such degradations may be necessary *prior to* any computations using the PSF in the image restoration postprocessing step. We use in publications [14,18] TV-based denoising of the PSF whose model is formulated as

$$s = h + \eta,$$

where h is the blur produced by the atmospheric turbulence on a single point, and s is the actual measurement or observed PSF. For the denoising problem, $H^*H = I$, so that the coefficient matrix in (13) is a sum of the identity matrix and $L_\beta(f^k)$. TV denoising of the PSF can be accomplished with little extra computation since point spread functions h for optical imaging can

usually be treated as having small extent. Thus, the cost of preprocessing the PSF is much less than that of deblurring f by TV methods. Again, our approach to the deblurring step is considered in publications [14] and [18], where numerically efficient and stable preconditioned iterative regularization methods are given.

GRA Paul Pauca was added to the grant in 1995. Mr. Pauca was actively involved with the research, including parallel programming support on the 400 node SP2. He was a co-author on two papers on image post-processing (see publications [14] and [18]), and has further helped by developing a Parallel Toolbox for MATLAB. This toolbox can also facilitate various adaptive-optics simulations. Information on the toolbox and the software system has been made available on the World Wide Web at:

http://www.cs.duke.edu/~pauca/research.html.

In addition Mr. Pauca is completing a paper on multichannel restoration for optical image data as a part of his current doctoral work.

Tests and analysis by the GRA on data provided by the Phillips Air Force Laboratory using our newly developed Multi-Level preconditioned conjugate gradient SVR method in combination with our TV-denoising scheme outlined above have given quite satisfactory results, and have been reported in publications [14] and [18]. The method is nonlinear, matrix-free and uses fast transform methods with a total order of $O(\ell n^2 \log n)$ where n^2 is the number of pixels in the image and $\ell << n$. Thus, our code can handle large-scale restorations with considerable savings in computation time. Comparisons were made with other restoration approaches, including the Total Variation (TV) method using nonlinear PDE models.

A sample restoration of a ground-based image of a satellite is given next. In this example a 256×256 image is considered. Specifically, the true object is an ocean reconnaissance satellite. A computer simulation algorithm was used to produce an image of the satellite, shown in Figure 2 (left), as it would be observed from a ground-based telescope using adaptive-optics compensation. The satellite was modeled as being 12 meters in length and in an orbit 500 kilometers above the surface of the earth. The charge-coupled device (CCD) for forming the image was a 65,536 pixel square array. CCD root-mean-square read-out noise variance was fixed at 15 microns per pixel to reflect a realistic state-of-the-art detector. Our computed restoration is shown in Figure 2 (right).

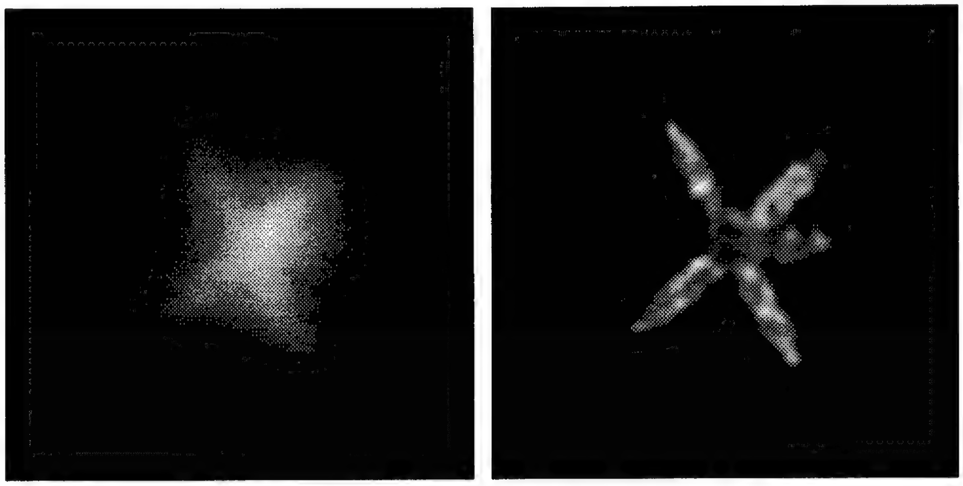

Figure 2: Degraded Image (left) and Restored Image (right).

4 Status of Effort at Expiration of Grant

Our research was concerned with several major projects in imaging. The first project concerned deformable mirror control in work with Dr. Brent Ellerbroek at the Air Force Phillips Laboratory. A second aspect of our research concerned nonlinear iterative methods for image post-processing computations. We are using the PI's account on the massively parallel SP2 at the AF Maui High Performance Computing Center for the parallel implementations. Our methods can handle large-scale restorations of Air Force data with considerable savings in computation time. We have further enhanced restored image quality by use of nonlinear optimization methods.

A recent phase of our work on image postprocessing involves the study of iterative blind deconvolution methods for image restoration. Blind deconvolution is the process of estimating both the true image and the blur from the degraded image characteristics, using partial information about the imaging system. Dr. Julian Christou from the AF Phillips Lab, Kirtland, reports in that "Experience with the adaptive-optics system at the Starfire Optical Range has shown that the point spread function (PSF) is non-uniform and varies both spatially and temporally as well as being object dependent. Because of this, the application of standard linear and nonlinear deconvolution algorithms make it difficult to deconvolve out the PSF in order to restore the image." This is especially significant in aero-optics postprocessing methods

where PSFs are generated using guide stars artificially generated from the back scatter of laser generated beacons. For blind image deconvolution we can write

$$g = h \star f + \eta, \tag{14}$$

where $\star$ denotes the convolution operator. The standard deconvolution problem is to recover the image f from (14), given the observed image g and the blurring operator h. Iterative blind deconvolution methods begin with a knowledge-based (positivity, finite support) estimate for f, "deconvolve" $g = h \star f$ to estimate h, and then iterate in an alternating fashion to improve the estimates for both f and h. Our purpose was to apply variational segmentation to the image and then to minimize the functional

$$||h \star f - g||^2 + \alpha_i J(f) + \beta_i J(h) \tag{15}$$

over both f and h. Here the parameters α_i and β_i were chosen based upon the level of smoothness in segment i. Both least squares and total variation norms were considered for the penalty operator $J(f)$, while the total variation norm will be used for defining the operator, $J(h)$. This choice is based in part on the small extent of the PSF h and its Gaussian form. Results thus far look very promising. The additional feature of estimating the PSF as well as deblurring the image in blind deconvolution adds another level of difficulty to the parallelization process, leading to some exciting challenges.

Activities for this grant included visits to Phillips Laboratory, Kirtland AFB, NM, and to Wright Laboratory, Wright-Patterson AFB, OH. Eighteen papers were published or submitted and twenty-eight presentations made during the 3 year grant period. An abstract including color images and graphics, "Leading Edge Methods in Optical Imaging", was prepared for the DOD publication *Success Stories in High Performance Computing.* In addition, testimony was prepared and presented in support of the FY 1997 Appropriations for the Department of Defense to the U.S. House of Representatives Subcommittee on National Security chaired by Representative C.W. Bill Young, House Committee on Appropriations, and to the U.S. Senate Subcommittee on Defense chaired by Senator Ted Stevens, Senate Committee on Appropriations.

A description of the status of this AFOSR work, including color images, graphics and multi-media animations, is on our Web page at:

http://www.mthcsc.wfu.edu/~plemmons/afres.

As we move toward the new millennium, the research results produced under this grant may very well have important impacts on science and engineering as part of a continuing development of the computational foundations of aero-optics technology. Packaging the results of our research into reliable software will further facilitate the effective and timely transfer of new knowledge gained here to DOD laboratories, other universities, and to industrial organizations. Some promising results and new ideas have been put forward and they indicate considerable potential for further progress in solving these important imaging problems in an efficient and stable way on modern computer architectures.

To summarize this Final Report, our research has concerned the major topics of 1) deformable mirror control computations in aero-optics, and 2) fast algorithms for image post-processing. Both areas demand sophisticated mathematics and state-of-the-art computation.

5 Personnel Supported by Grant

- PI: **Robert J. Plemmons**. Z. Smith Reynolds Professor, Wake Forest University.
- GRA: **Victor Paul Pauca**. Paul has completed his B.S. and M.S. degrees in Computer Science at Wake Forest University. He is enrolled in the Ph.D. program in Computer Science at Duke University, and has continued this work with PI Plemmons as part of his dissertation research.

6 Publications

Most of the papers listed below can be found on our Web page at:

http://www.mthcsc.wfu.edu/~plemmons/afres.

[1] B. Ellerbroek, C. Van Loan, N. Pitsianis, and R. Plemmons, *Optimizing closed loop adaptive optics performance using multiple control bandwidths*, **J. Optical. Soc. Amer.**, Vol. 11, November 1994, pp. 2871–2886.

[2] A. Berman and R. Plemmons, **Nonnegative Matrices in the Mathematical Sciences**, Classics in Applied Mathematics 9, SIAM Press, 1994.

[3] R. Chan, M. Ng, and R. Plemmons, *Preconditioners for atmospheric imaging*, Proc. **SPIE-95**, San Diego, Vol. 2296, pp. 528–539.

[4] J. Nagy, T. Torgersen, and R. Plemmons, *Fast restoration of atmospherically blurred images*, Proc. **SPIE-95**, San Diego, Vol. 2296, pp. 542–553.

[5] R. Chan, M. Ng, and R. Plemmons, *Generalization of Strang's preconditioner for Toeplitz least squares problems*, **Numerical Lin. Alg. and Applications**, Vol. 3, 1996, pp. 45–64.

[6] J. Nagy, T. Torgersen, and R. Plemmons, *Iterative image restoration using approximate inverse preconditioning*, **IEEE Trans. on Image Processing**, Vol. 5, 1996, pp. 1151–1162.

[7] M. Ng and R. Plemmons, *LMS-Newton adaptive filtering by FFT-based conjugate gradient iterations*, **Electronic J. Numer. Anal.**, Vol. 4, 1996, pp. 14–36.

[8] R. Plemmons, *Inverse problems in atmospheric imaging*, **Proc. Hellenic Inter. Conf. on Mathematics and Informatics**, Athens, Greece, 1996.

[9] B. Ellerbroek and R. Plemmons, *Computations in astro-imaging*, **Proc. IEEE Inter. Conf. on Image Processing**, Lausanne, Switzerland, 1996.

[10] M. Ng and R. Plemmons, *Fast RLS adaptive filtering by FFT-based conjugate gradient iterations*, **SIAM J. on Scientific Computing**, Vol. 17, July 1996, pp. 920–941.

[11] R. Plemmons, *Adaptive computations in optics*, **Proc. Institute for Math. Sci. Inter. Conf. on Mathematics in Signal Processing**, Warwick, Britain, 1996.

[12] B. Ellerbroek and R. Plemmons, *Leading edge methods in optical imaging*, **Success Stories in High Performance Computing**, U.S. Department of Defense Brochure, 1996.

[13] R. Plemmons, *Some applications of iterative deconvolution*, **SEA Bull. Math.**, Vol. 20, 1996, pp. 23-32.

[14] J. Nagy, P. Pauca, R. Plemmons and T. Torgersen, *Degradation reduction in optics imagery using Toeplitz structure*, preprint 1996, to appear **Proc. Cortona, Italy Conf. on Toeplitz Operators and Applications**, 1997.

[15] R. Plemmons, *Iterative numerical methods for imaging through turbulence*, to appear in **Proc. Conf. on Iter. Solution Methods for Scientific Comp.**, Nijmegen, The Netherlands, 1997.

[16] M. Ng, R. Plemmons and S. Qiao, *Regularized blind deconvolution using recursive inverse filtering*, **Proc. HK97 Conference on Scientific Computation**, Springer-Verlag, to appear 1997, full version submitted to **IEEE Trans. on Signal Processing**.

[17] B. Ellerbroek, C. Van Loan, N. Pitsianis, and R. Plemmons, *Jacobi-like method for a problem arising in adaptive-optics*, preprint 1996, to appear in **SIAM J. Optimization**, 1997.

[18] J. Nagy, P. Pauca, R. Plemmons and T. Torgersen, *Space-varying restoration of optical images*, preprint 1996, to appear in **J. Optical Soc. Amer.**, 1997.

7 Interactions and Transitions

7.1 Presentations

- Title: Least Squares Methods in Imaging
  Organization: AFOSR Grantees Conference
  Place: Phillips AF Lab., Albuquerque, NM
  Date: May, 1994

- Title: Computations in Imaging Restoration
  Organization: SIAM Conference
  Place: Salt Lake City, UT
  Date: June, 1994

- Title: Iterative Deconvolution
  Organization: SPIE International Conference
  Place: San Diego, CA
  Date: July, 1994

- Title: Some Computations in Atmospheric Imaging
  Organization: IFIPS International Conference, Plenary Talk
  Place: Raleigh, NC
  Date: November, 1994

- Title: Mathematics in Image Reconstruction and Restoration
  Organization: SEAS-SIAM Conference
  Place: Charleston, SC
  Date: March, 1995

- Title: Matrix Methods in Adaptive Optics
  Organization: Technion International Matrix Theory Conference
  Place: Haifa, Israel
  Date: May, 1995

- Title: Iterative Methods for Image Restoration
  Organization: Athens University, Colloquium Talk
  Place: Athens, Greece
  Date: June, 1995

- Title: Computations in Aero-Optical Imaging
  Organization: AFOSR Grantees Meeting

Place: Albuquerque, NM
Date: June, 1995

- Title: Iterative Deconvolution
  Organization: SPIE International Conference
  Place: San Diego, CA
  Date: July, 1995

- Title: Fast Algorithms for Optical Imaging
  Organization: AFOSR Workshop on Aero-Optics
  Place: Phillips Laboratory, Kirtland AFB, NM
  Date: July, 1995

- Title: Mathematics in Image Processing
  Organization: SIAM Annual Meeting
  Place: Charlotte, NC
  Date: October, 1995

- Title: Nonlinear and Space-Varying Methods in Image Reconstruction
  Organization: SIAM National Graduate Student Conference
  Invited Presentation by GRA Paul Pauca
  Place: Clemson, SC
  Date: April, 1996

- Title: Testimony on DOD FY 97 Appropriations for Mathematical Sciences Research
  Organization: U.S. House of Representatives Subcommittee on National Security
  Place: Washington, DC
  Date: May, 1996

- Title: Nonlinear and Adaptive Methods in Image Processing
  Organization: Workshop on Image Processing
  Place: Hong Kong, China (Chinese University of Hong Kong)
  Date: May, 1996

- Title: Computations in Aero-Optics
  Organization: AFOSR Grantees Meeting on Comp. and Phys. Math.
  Place: Wright Laboratory, Wright-Patterson AFB, OH
  Date: June, 1996

- Title: Testimony on DOD FY 97 Appropriations for Mathematical Sciences Research
  Organization: U.S. Senate Subcommittee on DOD Appropriations
  Place: Washington, DC
  Date: June, 1996

- Title: A Parallel Toolbox for MATLAB
  Organization: Albuquerque Resource Center, Univ. New Mexico
  Presentation by GRA Paul Pauca
  Place: Albuquerque, NM
  Date: July, 1996

- Title: Structured Problems in Optical Imaging
  Organization: Conf. on Structured Problems in Image and Signal Processing
  Place: Santa Barbara, CA
  Date: August, 1996

- Title: Inverse Problems in Optical Imaging
  Organization: Hellenic Inter. Conference on Scientific Computing
  Place: Athens, Greece
  Date: September, 1996

- Title: Blind Deconvolution
  Organization: University of Crete
  Place: Crete, Greece
  Date: October, 1996

- Title: Optical Imaging for Tracking Missiles
  Organization: Air Force Workshop on Airborne Laser Weapons
  Place: Albuquerque (Phillips AF Lab), New Mexico
  Date: October, 1996

- Title: Space Varying Regularization in Optical Imagery
  Organization: Scientific Computing Group, Computer Science Dept.
  Presentation by GRA Paul Pauca
  Place: Duke University, North Carolina
  Date: October, 1996

- Title: Computations in Optical Imaging
  Organization: IMA Conference on Signal Processing
  Place: Warwick, England
  Date: December, 1996

- Title: Numerical Linear Algebra in Optical Imaging
  Organization: Inter. Conference on Computational Mathematics
  Place: Rio de Janeiro, Brazil
  Date: January, 1997

- Title: A Matrix Optimization Problem in Adaptive Optics
  Organization: Workshop on Numerical Methods in Optimization
  Place: Curitiba, Brazil
  Date: January, 1997

- Title: Regularized Blind Deconvolution Using Recursive Filtering
  Organization: Workshop on Scientific Computing
  Place: Hong Kong, China
  Date: March, 1997

- Title: Optimization Methods in Aero-Optics
  Organization: Workshop on Optimization and Numerical Methods
  Place: Beijing, China
  Date: March, 1997

- Title: Large-Scale Computations in Optical Imaging
  Organization: SEAS-SIAM Annual Meeting
  Place: Raleigh, NC
  Date: April, 1997

7.2 Collaborative Research and Transitions at Air Force Laboratories

Activities for this grant included visits to Phillips Laboratory, Kirtland AFB, NM and to Wright Laboratory, Wright-Patterson AFB, OH. The PI has been in continuous contact with researchers at the Air Force Phillips Laboratory, Kirtland AFB, NM. The primary contact at Phillips Laboratory is Dr. Brent Ellerbroek at the Starfire Optical Range. His work is also supported by the AFOSR and we are collaborating on research involving a closed-loop

adaptive-optics system. In one paper the authors have helped to develop a theory with possible applications for closed-loop adaptive control methods to adjust the shape of these mirrors in real-time. A second paper has been completed. An abstract, "Leading Edge Methods in Optical Imaging", was prepared with Dr. Ellerbroek for the DOD publication Success Stories in High Performance Computing - 1996. Such collaboration is continuing, and a project involving minimal variance estimators in adaptive optics is in progress. Dr. Ellerbroek has also supplied us with Air Force satellite image data for tests with our image post-processing work. We are also interacting with Dr. Julian Christou and Dr. Donald Washburn at the AF PLK on blind deconvolution and the air borne laser weapons program, respectively. Contacts at Wright-Patterson AFB are Air Force Dr. Michael Roggemann and Dr. Byron Welsh.

Technology transfer of our work in relation to the projects at Air Force Laboratories included research on ground-based imaging of satellites, and related aero-optics activities. The PI has also participated in three AFOSR workshops at the Laboratories: 1) the *Aero-Optics and Image Reconstruction Workshop* concerning air borne laser weapons development (ABL), 2) the *Smart Sensors Workshop* concerning remote sensing for wide angle satellite surveillance, where the satellite systems will have on-board image processing capabilities when deployed, and 3) a second ABL Workshop concerning recent trends in missile tracking for the air born laser weapons program. Much of our work in this project concerned real-time adaptive filtering methods. Applications include closed-loop active noise (vibration) cancellation, with the potential for stabilizing firing pads for the air borne laser weapons.

In addition to the Air Force applications just described, potential technology transfer of our research on these imaging projects to civilian technology include *astronomical and medical imaging, and remote sensing for commercial purposes.* In astronomical imaging there is technology transfer to the Gemini 8-meter telescope international project and the astronomical community at large. Medical imaging applications of the adaptive-optics work include fluorescence microscopy in three dimensions, and the use of low energy laser beacons as an aid in deblurring images of the retina through the eyeball. Our image post-processing methods can also be applied to enhancing satellite images of the earth for agricultural, law enforcement, and geophysical purposes.

8 Inventions or Patent Disclosures

None. The research sponsored by the AFOSR under this grant concerned the development of rigorous mathematical models, computational algorithms and high performance computer software implementations that are readily available to the DoD and the private sector.